may also be used as shown in Fig. 8. It should be noted, however, that again, as in the case of σ_t versus life, as shown in Fig. 5, the smaller diameter ratios lie above the larger diameter ratios.

As can be noted from the similarity of correlation coefficients which are related to the spread of the data for the various cyclic parameters shown in Figs. 5 through 8, it makes little difference statistically as to what cyclic stress or strain parameter is chosen to plot the data. The magnitude of the data spread due to diameter-ratio dependence is approximately the same in each case with only the order being different. For the purpose of this report then, all data, unless otherwise specified, will be presented in terms of the normalized difference in principal bore stress as defined by equation (8).

Effects of Autofrettage on Fatigue Life. The effects of autofrettage on fatigue life, as compared to the nonautofrettaged condition, is shown in Figs. 9, 10, 11, and 12, respectively, for the diameter ratios of 1.4, 1.6, 1.8, and 2.0. A compilation of the least-squares lines for all diameter ratios in terms of the difference in principal bore stresses and cyclic pressure is shown in Figs. 13 and 14, respectively.

In the statistical data shown in the legend of these figures, S is the standard deviation as defined by

$$S = \sqrt{(1 - r^2) \frac{\Sigma (x - \bar{x})^2}{n - 2}}$$
(12)

and t_c is the confidence-level coefficient for a two-sided normal distribution which depends on the confidence level and the degrees of freedom defined as the number of test points minus two. In the figures, the values of t shown are for 99.9 percent and 99.0 percent confidence level. Coefficients for other confidence levels can be obtained from standard texts on statistics dealing with the treatment of experimental data [5, 6].

The limits for a given confidence band are closely approximated by the following relationship where \bar{x} is in \log_{10}

$$t_c = \bar{x} \pm t_c S \tag{13}$$

which represents a straight line parallel to the least-squares line on the curves presented. The relationship of cycles to failure to xis

$$N = (10^{x_e})$$
 (14)

For simplicity in using these curves, the value of D_c shown in the legend is the ratio of cycles to failure for the lower limit of con-



Journal of Basic Engineering

fidence level indicated to the least-squares value at a particular stress or pressure level. Since a two-sided normal distribution curve is assumed, the confidence level associated with the lower limit of the number of cycles to failure alone is increased by half the difference to unity for the values given. For example, the lower limit of life with 99.95 percent confidence is given by the relation

$$N_{99.95} = ND_{99.9} \tag{15}$$

As can be seen in the above-mentioned figures, there is an improvement in the fatigue characteristics of autofrettaged cylinders as compared to the nonautofrettaged condition. The relative benefit increases with decreasing operating stress level and increasing diameter ratio. The increase in life of the autofrettaged cylinders over the nonautofrettaged condition for several stress levels is summarized in Fig. 15. For example, considering the case of 2.0-diameter ratio operating at a normalized difference in principal stress of 0.9, which is approximately 10 percent below the elastic breakdown condition, as predicted by the von Mises yield criterion, the increase in life is a factor of 3.6. Proportional benefits are obtained in the allowable operating stresses to cause failure. Considering the same example, as above, for a life of 50,000 cycles, the average operating stress level, as a result of autofrettage, may be increased 50 percent over that for the nonautofrettaged condition.

Fig. 16 is a plot of diameter ratio versus cycles to failure for several differences in principal stress levels. As can be seen, there is a slight diameter-ratio dependency for the nonautofrettaged cylinders which is attributed primarily to the greater distance over which the crack must propagate as the diameter ratio increases, before ductile rupture occurs. It is readily seen, however, that the autofrettaged cylinders exhibit a very substantial diameter-ratio dependency with the benefit from autofrettage increasing with increased diameter ratio: From equation (3) this would be expected since the magnitude of the compressive residual bore stress increases with diameter ratio. In the region of 1.8 to 2.0-diameter ratio, the slope of the diameter ratio versus cycles to failure curve changes for the autofrettaged condition and approaches that characteristic of the nonautofrettaged cylinders. This indicates that the magnitude of the residual stresses is no longer increasing. However, by equating equation (3) to the yield strength of the material in compression, which is usually assumed substantially equal to that in tension, it can be shown that the maximum residual stress is obtained at a diameter ratio of 2.2, based on the Tresca yield criterion. To some extent, this early change in slope is attributed to the Bauschinger effect which, from associated work that will be reported at a later date, appears to occur at the 2.0-diameter ratio or less for the 100 percent overstrain condition. The Bauschinger effect results in a lowering of the compressive yield strength which, in the case of an





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